Theorems Featuring Facts of FFT Tables of Full Frequency

Sam Spiro, UC San Diego
(the one and only speaker of this talk)

Joint Work with G. Patchell and M. R. Thought
(both of whom are definitely real and neither of whom are talking today)

Outline

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This longstanding open problem was solved in the positive by Guldemond in 2020.

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For example, the following are (all of the non-isomorphic) grids giving 5 copies of the word FFT.

F	F	T
F	F	T
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F	F	T
F	F	T
F	T	T

F	F	T
F	F	T
T	T	T

T	F	F
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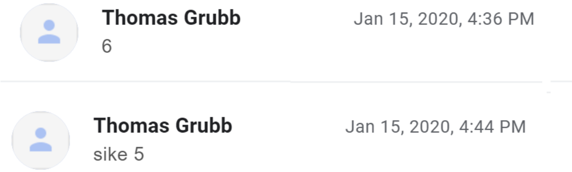

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For example, if w is the word consisting of n copies of the letter A and G is the $n \times n$ grid filled with the letter A, then $f(w, G) = 2n + 2 = f(w)$.

A	A	A
A	A	A
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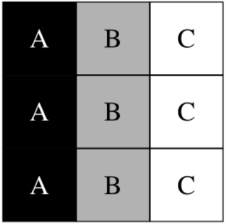

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The construction works by writing out w in row i whenever w_i=A, and then "trying to" write w in each column. For example, if w =BAACA we start with rows 2, 3, and 5.

B	A	A	C	A
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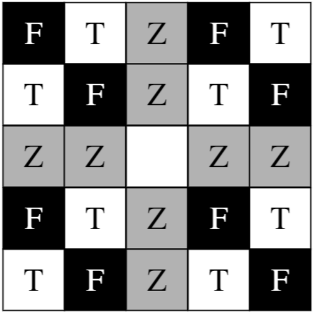

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Theorem (Patchell-Thought-S.; 2021)

Let $w = \mathrm{F}^{n-k}T^k$ be the word which is $n-k$ copies of F followed by k copies of T. If $1 \leq k \leq n/2$, we have

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For the lower bounds, if k is small then the letter F appears many times and we apply the "$2k+1$" construction, otherwise the word is very "antisymmetric" and we apply the $4k$ construction. For the upper bound, we break the argument into cases based on the number of diagonals achieved.

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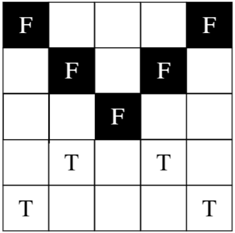
Say G contains two diagonals containing w. Then the first and last k rows can not be used.



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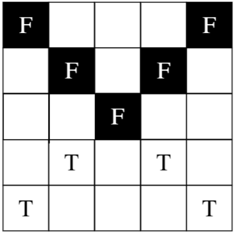

If none of the (middle) rows contain w, then

$$f(w, G) \leq n + 2.$$

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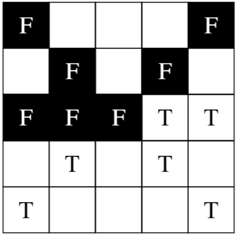

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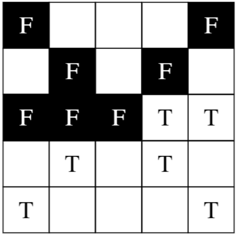

Thus counting rows, columns, and diagonals gives

$$f(w,G) \leq (n-2k)+(n-k)+2 = 2n-3k+2 \leq 2(n-k)+1. \qquad \square$$

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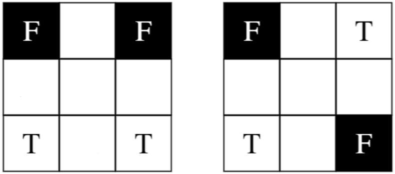
Consider the case $w_1 =$F and $w_n =$T. If two of the corners of G are labeled F and the other T, then this blocks two lines, so the best we can do is $2n$.



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The upper bound follows from the previous lemma, and the lower bound from the $4k$ anti-symmetric lemma (since there are $n/2$ positions where $w_i =$F and $w_{n-i+1} =$T).

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The lower bound follows $f(w) \geq n + 2$ (which holds for all w). This result is sharp: for infinitely many n there is a word such that each letter appears at most $1 + n/4$ with $f(w) > n + 2$.

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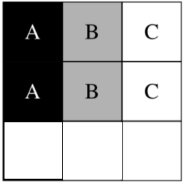

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where k is the maximum number of times a letter appears in w.

In particular, if no letter of w appears more than half the time, then the trivial lower bound $n + 2$ is correct.

Words in Grids

Corollary

Let w be the alternating word $\mathrm{FTFTFT}\cdots$. *Then*

$$f(w) = \begin{cases} n+3 & n \textit{ odd}, \\ 2n & n \textit{ even}. \end{cases}$$

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Let w be the alternating word $\mathrm{FTFTFT}\cdots$. *Then*

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In particular, $f(w)$ is very sensitive to the symmetries of w.

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Anyways, another natural question to ask is: what if we consider words of length k inside of an $n \times n$ grid? For example, the following 5×5 grid has 22 copies of the word CAT of length 3.

C	A	T	A	C
C	A	T	A	C
C	A	T	A	C
C	A	T	A	C
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More generally, we define $f(w, n)$ to be the maximum number of copies of the word w which can appear in an $n \times n$ grid.

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We essentially have no tight bounds for $f(w, n)$, though we can get surprisingly close in general.

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Theorem (Patchell-Thought-S.; 2021)

If w is a word of length k, then

$$(3n-4k)\cdot f(w,n,1) \leq f(w,n) \leq 2n\cdot f(w,n,1) + 4\sum_{i=k}^{n} f(w,i,1),$$

where $f(w,n,1)$ is the maximum number of times w can appear in a 1-dimensional grid of length n.

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If w is a word of length k, then

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where $f(w,n,1)$ is the maximum number of times w can appear in a 1-dimensional grid of length n.

Corollary

If $f(w,n,1) \sim \alpha n$ for some α, then

$$3\alpha n^2 \lesssim f(w,n) \lesssim 4\alpha n^2.$$

A Few Short Words on Short Words

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For the upper bound, there are at most $f(w, n, 1)$ copies of w in each of the $2n$ rows/columns, and you can also partition the diagonals into (at most) 4 1-dimensional lines of length i.

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If $f(w,n,1) \sim \alpha n$ *for some* α*, then*

$$3\alpha n^2 \lesssim f(w,n) \lesssim 4\alpha n^2.$$

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Our main question though is in general whether the lower or upper bounds of our corollary is closer to the truth.

A Few Short Words on Short Words

Conjecture

The lower bound of the previous corollary is correct for w the word on k distinct letters, i.e.

$$f(w, n) \sim \frac{3}{k-1} n^2.$$

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The lower bound of the previous corollary is correct for w the word on k distinct letters, i.e.

$$f(w, n) \sim \frac{3}{k-1} n^2.$$

The lower bound may even be correct for all w, but we're far from proving this. At one point I thought I had a heuristic proof solving this for $k = 2$, but we have no idea how to do this for $k = 3$ (the CAT problem).

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With Sam Spiro and Vaki Nikitopoulos I co-organize Food for Thought, the Graduate Student Seminar at UCSD.

Papers and Preprints:

1. On Optimizing Snack Selection. Journal of Snacks, 2019
2. A Proof of the Riemann Hypothesis, or True Implies True. Submitted, 2019.
3. The FFT Problem (with Gregory Patchell and Sam Spiro). The American Mathematical Monthly, Accepted 2021.